Drawing a surface in $\mathbb{L}^3$

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This paper is to introduce a known method of drawing minimal surfaces in $\mathbb{E}^3$ and show how to use it to find and draw a family of maximal surfaces in the Minkowski space $\mathbb{L}^3$.

1 Maximal surfaces in $\mathbb{E}^3$

The following table is a list of maximal surfaces in $\mathbb{L}^{\mathbf{3}}$.

Surfaces	**Riemann Surface M**	$\boldsymbol{f\,dz}$	$\boldsymbol{g}$
catenoid	$\mathbb{S}^2 \setminus \{0, \infty\}$	$\frac{1}{z^2}\,dz$	z
helicoid	$\mathbb{S}^2 \setminus \{0, \infty\}$	$\frac{i}{z^2}\,dz$	z
Enneper's surface	$\mathbb{C}$	dz	z
Trinoid	$\mathbb{S}^2 \setminus \{1, e^{2\pi i/3}, e^{4\pi i/3}\}$	$\frac{1}{(z^3-1)^2}\,dz$	z^2
Costa's surface	?	?	?

Table 1: A table

Minkowski space-time $\mathbb{L}^3$ is $\mathbb{R}^3 = \{(x, y, t)\}$ with pseudo-riemannian metric $dx^2 + dy^2 - dt^2$. Let M be a Riemann surface, and $f, g : M \to \mathbb{C}$ be analytic functions. Then, the Weierstrass-type formula

$$Re\left\{\int_{z_0}^{z} \big((1+g(w)^2)f(w), \boldsymbol{i}\,(1-g(w)^2)f(w), 2\,g(w)f(w)\big)\,dw\right\}$$

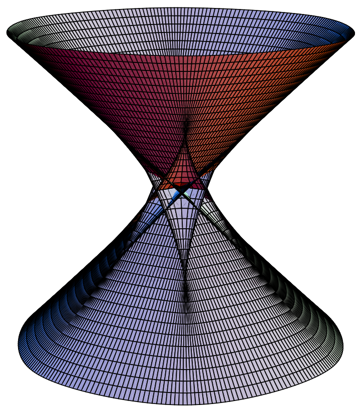

Figure 1: A figure

defines a space-like maximal immersion into $\mathbb{L}^3$.

For the periods of other components we have

Lemma 1. *The period around γ for the xy-components are 0 iff*

$$\sigma = \sqrt{\frac{1}{2}\frac{A}{B}}.$$

2 The Surface

We render the surface graphics using a 3D plotting function in Mathematica and we see the surface as above. (Figure 2)[1]

3 Final remarks

This gives a new surface which is in close relation with the Costa-Hoffman-Meeks minimal surface[1].

References

[1] Hoffman, D. and W. H. Meeks, III, Embedded minimal surfaces of finite topology, *Ann. of Math.* **131** (1990), 1–34.

[1] The figure number is not correct here.